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Heating of a weakly ionized magnetoplasma by oscillating electric field

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The heating of a weakly ionized magnetoplasma by electric field oscillations has been studied The Boltzmann transport equation in presence of external fields has been solved for obtaining the zero order and first order perturbations in the distribution function. The collisional relaxation of the perturbed distribution function has been invoked for the randomnization of the additional energy dumped into the plasma The energy balance equation has been solved in two different cases ($\boldsymbol{E} \perp \boldsymbol{B}$) and $\boldsymbol{E} \| \boldsymbol{B}$) and the rise in electron temperature obtained in each case. It is shown that if the electric field is applied in the form of impulses and by properly choosing plasma and field parameters the electron temperature can increase appreciably above the ambient temperature. The geophysical and astrophysical implications of the perturbational heating have been enumerated

1. INTRODUCTION

The electron velocity distribution function in presence of external forces has been studied by a number of authors. Holstein (1964) and Dreicer (1960) in their studies of the distribution function in presence of the electric field have taken account of the inelastic collisions as also the Coulumb interaction in the collision term of the Boltzmann transport equation. However, these authors do not consider the role of the magnetic field in their formulation. The behaviour of electron velocity distribution function in presence of electric and magnetic fields has been studied by Chapman & Cowling (1960). These authors restrict their formulation to the case of a sufficiently strong electric field oriented at right angles to a magnetic field. The fields considered were assumed to be uniform both in space and time. These restrictions were partially relaxed by Wu (1961) while considering the case of electric and magnetic fielods at any arbitrary orientation with respect to each other. The restriction regarding the uniformity of $\boldsymbol{E}$ and $\boldsymbol{B}$ was, however, not relaxed. Iso & Kamiyama (1969) extended the work of Wu by obtaining the distribution function in the case of a time varying electric field at any arbitrary angle to the steady magnetic field.

The effect of time varying electric field on a bounded and extended plasma has been the subject of considerable interest. The presence of VLF and ELF electric fields in the ionosphere and the magnetosphere has been established by Satellite probe measurements. The measurements carried out by Gurnett & Mozier (1969), Kelly *et al* (1970) and Taylor & Gurnett (1968) have shown their frequency and power spectra. Similar electric fields may exist in the solar corona in the form of self consistent electric fields induced by perturbed magneto-plasma. Many authors have studied the electric field interaction with plasma and have shown its effect on the velocity distribution function (Fain 1955, Kovrizhnykm 1960). The induced perturbation in the velocity distribution function depends on the nature of the interacting electric and magnetic fields and the ambient plasma. Sturrock (1966) and Puri (1966) have shown heating of the plasma by electric field impulses. A systematic kinetic analysis of heating of electron component of plasma by a time varying electric field was reported by Ginzburg & Gurevich (1960). Cronson & Wentzel (1963) accounted for non-linear effects of strong inhomogeneous and time varying electric field and have shown heating of the interacting plasma. The high frequency electromagnetic waves interacting with various plasma systems give rise to heating of the electron component (Pinskii 1964, Singh & Singh 1969). The high frequency heating of ionospheric plasma and study of cross modulation of a wanted signal has proved a very useful tool in the study of lower ionosphere (Singh 1964, Setty *et al* 1970, Nath & Setty 1974). The large high frequency power has been used recently for ionospheric modification experiment (Meltz 1973). This experiment has given interesting clue regarding heat balance in the lower ionosphere.

In this paper we have studied the problem of plasma heating by electric field oscillations. We have assumed the nature of electric field oscillations as $\boldsymbol{E} = \boldsymbol{E}_0 \cos \omega t$ and have obtained the isotropic part of the distribution function and the first order perturbation from the solution of the Boltzmann transport equation. We have also obtained the heating of an adopted model of laboratory plasma in two different cases one in which $\boldsymbol{E} \| \boldsymbol{B}$ and the other in which the two fields are transverse to each other. It has been shown that the heating is a very sensitive function of the amplitude of the oscillating electric field. We have made sample calculations and have shown that there can be significant heating of the electron component of the plasma due to this mechanism

3. Theoretical Formulation

In the presence of external forces the interacting electrons gain energy from the field and lose a small fraction of it to the heavier component of the ambient plasma during each elastic collision. Consequently the mean-square velocity of the electrons is much greater than the mean directed velocity. As a result of the impressed electric field the electron velocity distribution is perturbed. Under electrostatic approximation of the wave we ignore the effect of magnetic field vector. We will consider only small amplitude electric field oscillations and thus ignore the nonlinear perturbations in the velocity distribution function. In order to study the heating of a magnetoplasma in presence of a static magnetic field $\boldsymbol{B}$ and an oscillating electric field we suppose the plasma system to be spatially homogeneous. The Boltzmann transport equation can be written as

$$\frac{\partial f}{\partial t}+\boldsymbol{v}.\Delta f+\frac{q}{m_e}[\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}].\Delta f=\left(\frac{\partial f}{\partial t}\right)_{coll} \quad \ldots \quad (1)$$

In order to solve this equation for the first order perturbations in the distribution function we adopt the following perturbation scheme :

$$\boldsymbol{B}=\boldsymbol{B}_0$$

$$\boldsymbol{E}=\boldsymbol{E}_0\cos\omega t \quad \ldots \quad (2)$$

and

$$f(\boldsymbol{v},t)=f_0(v,t)+\frac{\boldsymbol{v}\cdot\boldsymbol{f}_1(v,t)}{v}$$

where $\boldsymbol{E}_0$ and ω are the amplitude and the angular frequency of the oscillating electric field, $f_0(v,t)$ is the symmetric part of the distribution function depending only on the modulus of velocity of $f_1(\boldsymbol{v},t)$ the asymmetric part arising due to external field. We will also assume the magnetic field to be acting in positive Z direction and the electric field acting at an angle β to the direction of the magnetic field. Substituting eq. (2) in eq. (1) we obtain the following system of equations satisfied by the symmetric and anisotropic part of the distribution function

$$\frac{\partial f_0}{\partial t}+\frac{q}{3m_ev^2}\frac{\partial}{\partial v}(v^2.\boldsymbol{E}\cdot\boldsymbol{f}_1)-\frac{1}{v^2}\frac{\partial}{\partial v}\left\{\frac{\nu v^2kT_0}{M}\frac{\partial f_0}{\partial v}+\frac{\nu v^3m_ef_0}{M}\right\}=0 \quad \ldots \quad (3)$$

and

$$\frac{\partial \boldsymbol{f}_1}{\partial t}+\frac{q\boldsymbol{E}}{m_e}\frac{\partial f_0}{\partial v}+\frac{q}{m_e}[\boldsymbol{B}_0\times\boldsymbol{f}_1]+\nu\boldsymbol{f}_1=0. \quad \ldots \quad (4)$$

In writing eq. (3) the collisional term on the right hand side of the Boltzmann transport equation has been replaced by Desloge and Matthysee collision term whereas the BGK model has been adopted in approximating the collision term in eq. (4). Solving eqs. (3) and (4) we obtain the following expression for the asymmetric and the symmetric parts of the distribution function.

$$\boldsymbol{f}_1=\frac{-qE_0}{2m_e}\left(\frac{\partial f_0}{\partial v}\right)\left[\frac{\left\{(\nu+i\omega)^2\boldsymbol{E}_0/E_0+\omega_{ce}^2\cos\beta\boldsymbol{B}_0/B_0+\omega_{ce}(\nu+i\omega)\frac{\boldsymbol{B}_0\times\boldsymbol{E}_0}{B_0E_0}\right\}e^{i\omega t}}{(\nu+i\omega)\{\omega_{ce}^2+(\nu+i\omega)^2\}}\right.$$

$$\left.+\frac{\left\{(\nu-i\omega)^2\boldsymbol{E}_0/E_0+\omega_{ce}^2\cos\beta.\boldsymbol{B}_0/B_0+\omega_{ce}(\nu-i\omega)\frac{\boldsymbol{B}_0\times\boldsymbol{E}_0}{B_0E_0}\right\}e^{-i\omega t}}{(\nu-i\omega)\{\omega_{ce}^2+(\nu-i\omega)^2\}}\right] \quad \ldots \quad (5)$$

and
$$f_0 = D\exp\left\{-\int_0^v m_e v\,dv/kT_0 + \frac{q^2E_0{}^2\phi}{3m_e\delta}\right\} \tag{6}$$

where
$$\phi = \frac{\cos^2\beta}{\omega^2+\nu^2} + \frac{\sin^2\beta}{2}\left[\frac{1}{(\omega+\omega_{ce})^2+\nu^2}+\frac{1}{(\omega-\omega_{ce})^2+\nu^2}\right] \tag{7}$$

δ, is the energy loss parameter and ν represents the electron neutral particle collision frequency. In order to keep the problem easily tractable and consistent with the perturbation scheme we take the collision frequency as velocity independent. With this assumption eq. (6) reduces to

$$f_0 = D\exp\left[\frac{-m_e v^2/2}{kT_0 + \frac{q^2E_0{}^2\phi}{3m_e\delta}}\right]. \qquad \ldots \quad (8)$$

The constant D appearing in eq. (8) is determined from the normalization condition

$$4\pi\int_0^\infty f_0 v^2 dv = N_e \tag{9}$$

where N_e is the electron density. Eq. (9) with the help of eq. (8) is written as

$$4\pi D\int_0^\infty v^2\exp(-pv^2)dv = N_e \qquad \ldots \quad (10)$$

or,
$$D = N_e(p/\pi)^{3/2} \tag{11}$$

where
$$p = \frac{\ }{2[kT_0+(q^2E_0{}^2\phi)/3m\delta_e]}\,. \tag{12}$$

The symmetric part of the distribution function is thus written as

$$f_0 = N_e(p/\pi)^{3/2}\exp(-pv^2). \tag{13}$$

Differentiating eq. (13) with respect to v we obtain

$$\partial f_0/\partial v = -\left(\frac{2N_e}{\pi^{3/2}}\right)p^{5/2}v\exp(-pv^2). \tag{14}$$

Substituting eq. (14) in eq. (5) the exact solution of the Boltzmann transport equation is obtained.

Heating of electron component of the plasma

The velocity distribution function perturbed by the presence of oscillating electric field relaxes to its unperturbed value after a finite time. The relaxation of the distribution function and the energy dissipated in the plasma system is chiefly governed by collisional term chosen for the plasma system. In this case we have not assumed any form of the isotropic part of the velocity distribution and instead we have solved eq. (3) to obtain the form of f_0. The energy balance equation for this system is written as

$$\frac{d\epsilon}{dt} = \int_0^\infty (\partial f/\partial t)_{coll}\cdot\tfrac{1}{2}mv^2.dv^3. \qquad \ldots \quad (15)$$

We proceed to solve this equation by replacing the collisional term by $-\nu f_1$ and the element of volume in velocity space by $4\pi v^2 dv$. The form of f_1 is further simplified by considering the electric and magnetic field directions. We consider two specific cases.

Case $\boldsymbol{B} \| \boldsymbol{E}$

In this case $\beta = 0$ so that eqs. (5) and (7) can be written as

$$f_1 = \frac{-qE_0}{2m_e}\frac{\partial f_0}{\partial v}\left[\frac{\left\{(\nu+i\omega)^2\frac{\boldsymbol{E}_0}{E_0}+\omega_{ce}^2\frac{\boldsymbol{B}_0}{B_0}\right\}e^{i\omega t}}{(\nu+i\omega)\{\omega_{ce}^2+(\nu+i\omega)^2\}}+\frac{\left\{(\nu-i\omega)^2\frac{\boldsymbol{E}_0}{E_0}+\omega_{ce}^2\frac{\boldsymbol{B}_0}{B_0}\right\}e^{-i\omega t}}{(\nu-i\omega)\{\omega_{ce}^2+(\nu-i\omega)^2\}}\right] \quad \ldots (16)$$

and

$$\phi = \frac{1}{\omega^2+\nu^2} \quad \ldots (17)$$

We now substitute for $\frac{\partial f_0}{\partial v}$ in eq. (16) to obtain

$$f_1 = \frac{2N_e qE_0 p^{5/2} v \exp(-pv^2)}{m_e \pi^{3/2}}\left\{\frac{\nu\cos\omega t+\omega\sin\omega t}{\nu^2+\omega^2}\right\} \quad \ldots (18)$$

Substituting for f_1 in eq. (15) and carrying out the indicated integrations we obtain

$$\frac{d\epsilon}{dt} = -\frac{4\nu qE_0 N_e p^{-\frac{1}{2}}(\nu\cos\omega t+\omega\sin\omega t)}{\pi^{\frac{1}{2}}.(\omega^2+\nu^2)}. \quad \ldots (19)$$

Replacing the energy density by $3/2N_eKT_e$ and integrating eq. (19) with the initial condition that

$$T_e = T_0 \text{ at } t = 0$$

we get

$$(T_e-T_0) = \left(\frac{2}{m_e.\pi}\right)^{\frac{1}{2}}\frac{8\nu|q|E_0}{3k(\omega^2+\nu^2)}\left[kT_0+\frac{q^2E_0^2\phi}{3m_e\delta}\right]^{\frac{1}{2}}\left[\frac{\nu}{\omega}\sin\omega t+1-\cos\omega t\right] \quad \ldots (20)$$

Case II. $\boldsymbol{E} \perp \boldsymbol{B}$

In this case $\beta = q_0^0$ therefore, from eq. (5) we get :

$$\boldsymbol{f}_1 = -\left(\frac{qE_0}{2m_e}\right)\frac{\partial f_0}{\partial v}\left[\frac{\left\{(\nu+i\omega)^2\frac{\boldsymbol{E}_0}{E_0}+\omega_{ce}(\nu+i\omega)\frac{\boldsymbol{B}_0\times\boldsymbol{E}_0}{B_0E_0}\right\}e^{i\omega t}}{(\nu+i\omega)\{\omega_{ce}^2+(\nu+i\omega)^2\}}\right.$$

$$\left.+\frac{\left\{(\nu-i\omega)^2\boldsymbol{E}_0/E_0+\omega_{ce}(\nu-i\omega)\frac{\boldsymbol{B}_0\times\boldsymbol{E}_0}{B_0E_0}\right\}e^{-i\omega t}}{(\nu-i\omega)\{\omega_{ce}^2+(\nu-i\omega)^2\}}\right] \quad \ldots (21)$$

Eq. (21) on further simplification can be written as

$$f_1 = \frac{-qE_0}{2m_e}\left(\frac{\partial f_0}{\partial v}\right)\left[\frac{e^{i\omega t}}{\{(\nu+i\omega)^2+\omega_{ce}^2\}^{\frac{1}{2}}}+\frac{e^{-i\omega t}}{\{(\nu-i\omega)^2+\omega_{ce}^2\}^{\frac{1}{2}}}\right] \quad \dots \quad (22)$$

Putting $\beta = 90°$ in eq. (7) we get

$$\phi = \tfrac{1}{2}\left[\frac{1}{(\omega+\omega_{ce})^2+\nu^2}+\frac{1}{(\omega-\omega)_{ce}^2+\nu^2}\right] \quad \dots \quad (23)$$

Substituting for $\left(\frac{\partial f_0}{\partial v}\right)$ from eq. (14) we rewrite eq. (22) as

$$f_1 = \frac{qE_0N_e v p^{5/2}\exp(-pv^2)}{m_e\pi^{3/2}}\left[\frac{(\cos\omega t+i\sin\omega t)}{\{\omega_{ce}^2+(\nu+i\omega)^2\}^{\frac{1}{2}}}+\frac{(\cos\omega t-i\sin\omega t)}{\{\omega_{ce}^2+(\nu-i\omega)^2\}^{\frac{1}{2}}}\right] \quad \dots \quad (24)$$

The square brace in eq. (24) is the sum of a complex quantity and its complex conjugate which, therefore, is real and equals twice the real part of each term. Therefore, eq. (24) is written as

$$f_1 = \frac{2qE_0N_e v p^{5/2}\exp(-pv^2)}{m_e\pi^{3/2}}\left[\frac{A_1\cos\omega t+B_1\sin\omega t}{(\alpha'^2+\beta'^2)^{\frac{1}{2}}}\right] \quad \dots \quad (25)$$

where

$$\alpha' = \omega_{ce}^2+\nu^2-\omega^2$$

$$\beta' = 2\nu\omega$$

$$A_1 = \left[\frac{\alpha'+(\alpha'^2+\beta'^2)^{\frac{1}{2}}}{2}\right]^{\frac{1}{2}} \quad \dots \quad (26)$$

and

$$B_1 = \beta'/2\left[\frac{\alpha'+(\alpha'^2+\beta'^2)^{\frac{1}{2}}}{2}\right]^{-\frac{1}{2}}$$

With the help of eq. (25) we write the energy balance equation as

$$\frac{d\epsilon}{dt} = \frac{-4N_e\nu qE_0p^{5/2}}{(\alpha'^2+\beta'^2)^{\frac{1}{2}}.\pi^{\frac{1}{2}}}\{A_1\cos\omega t+B_1\sin\omega t\}\int_0^\infty v^5\exp(-pv^2)dv$$

$$= \frac{-4N_e\nu qE_0(A_1\cos\omega t+B_1\sin\omega t)}{\pi^{\frac{1}{2}}p^{\frac{1}{2}}(\alpha'^2+\beta'^2)^{\frac{1}{2}}} \quad (27)$$

substituting for ϵ and p eq. (27) can be written as

$$\frac{3}{2}N_ek\frac{dT_e}{dt} = \frac{-4\sqrt{2}N_e\nu qE_0[A_1\cos\omega t+B_1\sin\omega t]\left[kT_0+\frac{q^2E_0^2\phi}{3m_e\delta}\right]^{\frac{1}{2}}}{\pi^{\frac{1}{2}}m_e^{\frac{1}{2}}(\alpha'^2+\beta'^2)^{\frac{1}{2}}} \quad (28)$$

Integrating eq. (28) with the same initial condition as used in the integration of eq. (19) we obtain

$$(T_e - T_0) = \frac{8\sqrt{2\nu}\,|q|\,E_0\left[kT_0 + \frac{q^2 E_0^{\,2}\phi}{3m_e\delta}\right]^{\frac{1}{2}}[A_1 \sin \omega t + B_1 - B_1 \cos \omega t]}{3\omega k \pi^{\frac{1}{2}} m_e^{\frac{1}{2}}(\alpha'^2 + \beta'^2)^{\frac{1}{2}}} \quad \ldots \quad (92)$$

3. Results and Discussion

If the plasma and field parameters are properly chosen the temperature of the electron component would increase substantially as compared to the ambient thermal equilibrium temperature. Therefore, the plasma system subjected to electric field oscillations invariably gives rise to thermal non-equilibrium. We make an estimate of the heating produced using eqs. (20) and (29). The following plasma and field parameters have been chosen.

$\nu = 10^6$ rad-sec^{-1}	$T_0 = 10^4$ °K
$\omega = 10^6$ rad-sec^{-1}	$B = 10^{-4}\ W/m^2$
$t = 10^{-6}$ sec	$\delta = 10^{-3}$

Taking $E_0 = 0{\cdot}15$ V/m, the electron temperature computed from eqs. (20) and (29) comes out to be $1{\cdot}1254\ T_0$ and $1{\cdot}086\ T_0$ respectively. The electron temperature is found to be strongly correlated with the perturbation time and varies harmonically with it. If the perturbation time is properly selected and the electric field applied in the form of impulses it is likely that the electron temperature may increase appreciably above the ambient temperature. Further in the case of an actual laboratory plasma the problem of its containment for eliminating the surface effects due to high temperature plasma coming in contact with the container surface is solved by using a steady magnetic field. We find that the heating of the plasma also depends upon the orientation of the electric field with respect to the magnetic field and on the amplitude of the electric field impulse. Although at higher values of the electric field amplitude the non linear effects will have to be taken care of yet we conclude that the plasma can be heated effectively by large amplitude electric field in the presence of a confining magnetic field. From eqs. (20) and (29) we find that theh eating should be more effective corresponding to lower frequencies of the oscillating electric field. However, a closer scrutiny of these equations shows that this effect is masked by large collision frequency which controls the energy transfer mechanism in the case of a collision dominated plasma.

We find that electric field perturbations and oscillations in ionospheric and magnetospheric plasma are one of the important sources for localized heating of plasma. The localized heating thus controls the small scale motion of the plasma. The electric field perturbations and oscillations would indeed be a very important soure of heating of solar coronal plasma. With careful choice of plasma system and field parameters thermal non-equilibrium plasma can be produced in the laboratory.

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